1.7 Solve Problems Using Guess and Test

GOAL

Solve problems involving rational numbers using a guess-and-test strategy.

LEARN ABOUT the Math

Eight students rated their video game enjoyment on a scale from −10 to 10.

- The mean score was 1.625.
- Three scores were negative and added to −7.
- Two positive scores were double two other ones.
- There were no negative doubles.

? What were the eight scores?

EXAMPLE 1 | Using guess and test to determine data values

Thomas's Solution

1. Understand the Problem

I know the numbers all have to be between −10 and 10.
I know there have to be eight numbers.
I know the mean of the eight numbers is 1.625.
I know that three numbers are negative and that they add to −7.
I know that exactly two numbers are the double of two other ones and they are positive.

2. Make a Plan

I will figure out the total of all the numbers, and then the total of the positive numbers.
I will figure out what the five positive numbers add to. Then, I will make sure that they add to the correct total but also make sure that two numbers are double the other two.

3. Carry Out the Plan
Since the mean is based on 8 numbers,

Total of all 8 numbers $= 8 \times 1.625 = 13$

Total of negatives $= -7$
Total of positives $+ (-7) = 13$
Total of positives $= 20$

First, I tried 8 and 4 and 6 and 3 as the doubles:
$8 + 4 + 6 + 3 +$ another positive $= 20$
This is not possible since $8 + 4 + 6 + 3$ is already 21.

Then, I tried 6 and 3 and 2 and 1 as the doubles:

$20 - (6 + 3 + 2 + 1) = 20 - 12 = 8$

These values worked because 8 is not the double of any other positive. The positives are 6, 3, 2, 1, and 8.
The negatives had to add to -7. I tried -1, -2, and -4, but -4 is double -2 and there can be no negative doubles.
Next, I tried -1, -1 and -5.

My solution is that the scores are 3, 6, 1, 2, 8, -5, -1, and -1.

4. Look Back
I checked by adding and dividing by 8 to make sure that the mean was correct.
$[3 + 6 + 1 + 2 + 8 + (-5) + (-1) + (-1)] \div 8 = 1.625$

Reflecting

A. How did Thomas use logical reasoning in solving the problem?

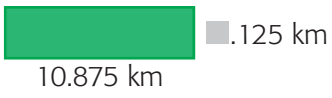
B. How did using guess and test help Thomas to solve the problem?

C. Could Thomas have found a different solution? Explain.

WORK WITH the Math

EXAMPLE 2 Using guess and test to solve a measurement problem

The number of kilometres in the perimeter of this park is about 6 less than the number of square kilometres in its area. What is the width?



Sam's Solution

1. Understand the Problem
I have to determine the width so that the number of units in the perimeter is about 6 less than the number of units in the area.

2. Make a Plan
I will use estimates for the width and then determine the perimeter and the area.

3. Carry Out the Plan
First, I estimated the length as 11 km.
I tried a width of 5 km.

The perimeter is about $2 \times 16 = 32$ km.
The area is about $5 \times 11 = 55$ km^2.
$55 - 32 = 23$
So, the difference is too much.

I tried a width of 3 km.
The perimeter is about $2 \times 14 = 28$ km.
The area is about $3 \times 11 = 33$ km^2.
$33 - 28 = 5$
The difference is not quite enough, but it is close.

The diagram said the width was actually ■.125 km and the length was actually 10.875 km. Since I was so close, I used 3.125 km.
Perimeter $= 2 \times 14.000 = 28.000$ km
Area $= 3.125 \times 10.875 = 33.984$ km^2 (rounded to the nearest thousandth)

The difference is 5.984, rounded to the nearest thousandth. This is very close to 6. Therefore, the width is 3.125 km.

4. Look Back
I checked the perimeter and area for widths of 2.125 km and 4.125 km. For 2.125 km, perimeter $= 24$ km and area $\doteq 23$ km^2. For 4.125 km, perimeter $= 32$ km and area $\doteq 45$ km^2. So, the missing digit could only be 3.

In Summary

Key Idea

- You can use guess and test in combination with other strategies to help you solve problems. Once you guess, you can use the results of your guess to improve your next guess.

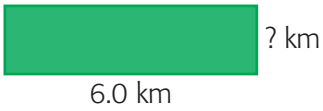
Checking

1. You subtract a rational number from its triple. The result is $-\frac{5}{8}$. What is the rational number?

Practising

2. The mean of five numbers is -5. The sum of the positive numbers in the set is 37 greater than the sum of the negative numbers in the set. What could the numbers be?

3. The number of kilometres in the perimeter of this park is 8 greater than the number of square kilometres in its area. What is the width? Use a calculator to help.



4. The sum of three rationals is 0. Two have denominators of 8. The product of two of them is $\frac{1}{16}$. The quotient of two of them is -5. What are the rationals?

5. The product of two opposites is -1.8225. What are the numbers? Use a calculator to help you guess and test.

6. The sum of five numbers is $-\frac{1}{4}$. The numbers include two pairs of opposites. The quotient of two values is 2. The quotient of two different values is $-\frac{3}{4}$. What are the values?

7. A share price increased in value one day, doubled that increase the next day, and then decreased \$0.12 in value the following day. If the total change was \$0.33, by how much did the share go up the first day?

8. A rational number with a denominator of 9 is divided by $\left(-\frac{2}{3}\right)$. The result is multiplied by $\frac{4}{5}$ and then $-\frac{5}{6}$ is added. The final value is $\frac{1}{10}$. What was the original rational?

9. Two rational numbers are added. The sum is $\frac{1}{4}$ more than the product. What are the rational numbers?

Closing

10. Why is guess and test a good strategy to use for questions like question 3? Why might you also work backward?

Extending

11. Create a problem involving rational numbers that would be reasonable to solve with a guess-and-test strategy. Then, solve it using that strategy.